Applying the Economic Order Quantity Formula to Determine Optimal Trade Size for an Equity Program

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Abstract

The original Economic Order Quantity (EOQ) model is typically encountered in management science courses and is concerned with minimizing total ordering costs associated with inventory replenishment. It turns out that the original EOQ variables corresponding to quantity demanded, ordering cost, and unit inventory cost can be reinterpreted in the context of how to execute an equity trading program that minimizes total execution cost. The reinterpreted variables become total shares, trading cost, and per share market impact cost, respectively. The cases where per share market impact cost is fixed and variable are covered.

1 Introduction

Anyone who has taken a management science course has probably seen the Economic Order Quantity (EOQ) formula[1,2]. EOQ is arrived at by differentiating the Total Cost (TC) function with respect to Quantity (Q), setting the result to zero, and solving for the optimal order quantity Q* that minimizes TC. It turns out that one can easily recast the formula's original input variables in terms their corresponding equity order cost variables in order to arrive at $\hat{Q}$*, which is the optimal order size for executing a large equity share order buy or sell program. Similar to how the original EOQ formula has historically been used to determine the least cost order size for inventory reordering, the modified EOQ which we'll call $\widehat{EOQ}$, calculates the least cost equity order size that minimizes total trading costs for a given trading program. The advantages of $\widehat{EOQ}$ are similar to that of EOQ, mainly simplicity and tractability.

Like its EOQ counterpart where the key drivers are ordering and holding costs, the key drivers of $\widehat{EOQ}$ are fixed ordering costs, which we'll call trading cost, and per unit costs driven by size which we'll call market impacts cost. Trading cost typically consists of commissions, but additional costs can be exchange fees, exercise fees, clearing fees, taxes, and miscellaneous fees[3]. Like ordering costs, trading costs are typically fixed and or generally can be fully determined before placing a trade. On the other hand, market impact cost[4] is the extent to which the act of buying or selling moves the price against the trader, i.e. upward when buying and downward when selling, and generally cannot be known precisely although estimation techniques exist. Market impact can arise because the price needs to move sufficiently to tempt other investors to buy or sell. It can arise because professional investors may position themselves to profit from knowledge that a large investor is active in the market one way or the other. Furthermore, some financial intermediaries have such low trading costs that they can profit from price movements too small to be of relevance to a majority of investors.

The measurement of market impact cost is beyond the scope of this paper and will be assumed as either constant or variable in the examples that follow, but in its most basic form market impact cost can be thought of as the absolute change in price divided by volume[5]. The financial institution seeking to limit its market impact cost consistent with full program execution will in general need to limit the pace of its trading activity so as to minimize any adverse disruption to price. However, once market impact cost has been estimated the application of $\widehat{EOQ}$ can assist in managing it.

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[1] Harris, Ford W. "How Many Parts To Make At Once" Factory, The Magazine of Management, 10(2), 135-136, 152 (1913).

[2] Wilson, R. H. "A Scientific Routine for Stock Control" Harvard Business Review, 13, 116-128 (1934).

[3] Certificate fees, Postage and Handling.

[4] For those wanting to know more on how market impact costs can be measured please refer to the excellent papers by Torre and Ferrari (1999) and Almgren, Thum, Hauptmann, and Li (2005).

[5] For example, Kyle's lambda named from Albert Kyle's famous paper on market microstructure (1985).

For example, when a trader needs to execute a relatively large share purchase program (say 10% or more of average daily trading volume) he will want to minimize total execution costs (trading cost + market impact cost). At one extreme trading too many shares, say, buying all shares at once can create liquidity issues and adversely move price against the trade and thus increase market impact cost. On the other hand, trading more slowly by trading fewer shares and increasing the number of orders will drive up trading costs such as commissions. As with the original EOQ, $\widehat{EOQ}$ attempts to minimize the two cost drivers, trading cost and market impact cost, which tend to move in opposite directions relative to total number of trades placed. Before seeing how $\widehat{EOQ}$ can assist in helping to minimize total execution cost, a review of EOQ is presented next.

2 EOQ Model Overview

For those not familiar with the original single-item EOQ model, the assumptions/variables are follows.

EOQ Assumptions:

- Ordering cost is constant.
- Rate of demand is known, and spread evenly throughout the year.
- Lead time is fixed.
- Purchase price of the item is constant i.e. no discount is available
- Replenishment is made instantaneously; the whole batch is delivered at once.
- Only one product is involved.

EOQ Variables:

- Q = order quantity.
- Q* = optimal order quantity.
- D = annual demand, in units.
- S = fixed cost per order (not per unit, typically cost of ordering and shipping and handling, and is not the cost of goods).
- H = annual holding cost per unit (also known as carrying cost or storage cost) (warehouse space, refrigeration, insurance, etc. usually not related to the unit cost).
- P = price per unit.

The EOQ formula finds the minimum point of the following total cost function, TC:

- TC = purchase cost + ordering cost + holding cost
- Purchase cost - This is the variable cost of goods: Purchase unit price times annual demand quantity, or $P \cdot D$.
- Ordering cost - This is the cost of placing orders; each order has a fixed cost S, and we need to order D/Q times per year. This is $D/Q \cdot S$.
- Holding cost: the average quantity in stock (between fully replenished and empty) is Q/2, so this cost is $H \cdot Q/2$.
- Thus, the Total Cost function is $TC = P \cdot D + D/Q \cdot S + H \cdot Q/2$.

EOQ is the quantity Q* to order, so that ordering cost plus inventory carrying cost is at its minimum on the TC curve. To determine Q*, set the ordering cost equal to the holding cost. Alternatively, one can start with the TC formula and apply some basic calculus as follows.

- Differentiate TC with respect to Q: $\frac{\partial TC}{\partial Q} = -D \cdot S/Q^2 + H/2$.
- Set above equal to zero: $-D \cdot S/Q^2 + H/2 = 0$.
- Solve for the optimal Q, Q*: $Q^2 = 2 \cdot D \cdot S/H$.

Thus, $Q* = \sqrt{2 \cdot D \cdot S/H} = EOQ.$ \hfill (1)

3 Specializing EOQ to Arrive at $\widehat{EOQ}$

Starting with the original EOQ assumptions and variables as described above we can recast them in terms of executing an equity trading program as follows.

$\widehat{EOQ}$ Assumptions:

- Commissions plus specific fees, per order, are constant.

- Total number of shares to execute in the program and expected time horizon for completion are known.

- Time between placing an order and its execution is fixed, at least approximately.

- Trader agrees to pay a limit price or some type of average price (i.e. VWAP[6]) over the course of the program.

- Each order over the course of the program is fully executed (i.e. no partial or unfilled orders)

- Only one stock is being traded during the program.

$\widehat{EOQ}$ Variables:

- $\hat{Q}$ = equity order quantity, in shares.

- $\hat{Q}^*$ = optimal equity order quantity, in shares.

- $\hat{D}$ = total number of shares to be traded.

- $\hat{S}$ = trading cost such as fixed commissions and fees, per equity order.

- $\hat{H}$ = market impacts cost, per share.

- $\hat{P}$ = stock price, per share.

Thus, the total cost function is $\widehat{TC} = \hat{P} \cdot \hat{D} + \hat{P}/\hat{Q} \cdot \hat{S} + \hat{H} \cdot \hat{Q}/2$. In similar fashion to how EOQ was defined,

$$\widehat{EOQ} = \sqrt{2 \cdot \hat{D} \cdot \hat{S}/\hat{H}} = \hat{Q}^*. \tag{2}$$

Now that everything has been defined, it's time to see a couple worked out examples. In what follows we'll consider the case where market impacts cost is constant and where it's variable.

4 Managing Execution Cost with $\widehat{EOQ}$

To start, let's assume we're a large investment bank needing to execute a sizeable order in IBM for a customer. The customer wants to buy one million shares and would like to receive some sort of average market price for the trading day. The latest average 5-day volume is 4.5 million shares and there are 1.16 billion shares outstanding, so about 0.39% of total shares turns over daily. Trading cost is $2.50 per order and market impact cost has been estimated to be, on average, $0.05 per share. These details are summarized in table 1.

Table 1	
Company:	IBM
Average Daily Volume:	4,500,000
Total Shares to Execute:	1,500,000
Market Impact Cost/Share:	$0.20
Fixed Cost/Order:	$2.50
Average Price/Share:	$172.00

Plugging the values into (2) gives $\widehat{EOQ} = \sqrt{2 \cdot 1,000,000 \cdot 2.50/0.20} = 5,000$.

In other words, buying one million shares of IBM, five thousand shares at a time over the course of a trading day, will minimize total cost and take two hundred orders to accomplish (1,000,000 shares divided by 5,000 shares per order). We can observe graphically how the cost curves behave, and where the minimum cost occurs, in figure 1.

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[6]VWAP is the ratio of the value traded to total volume traded over a particular time horizon (usually one day).
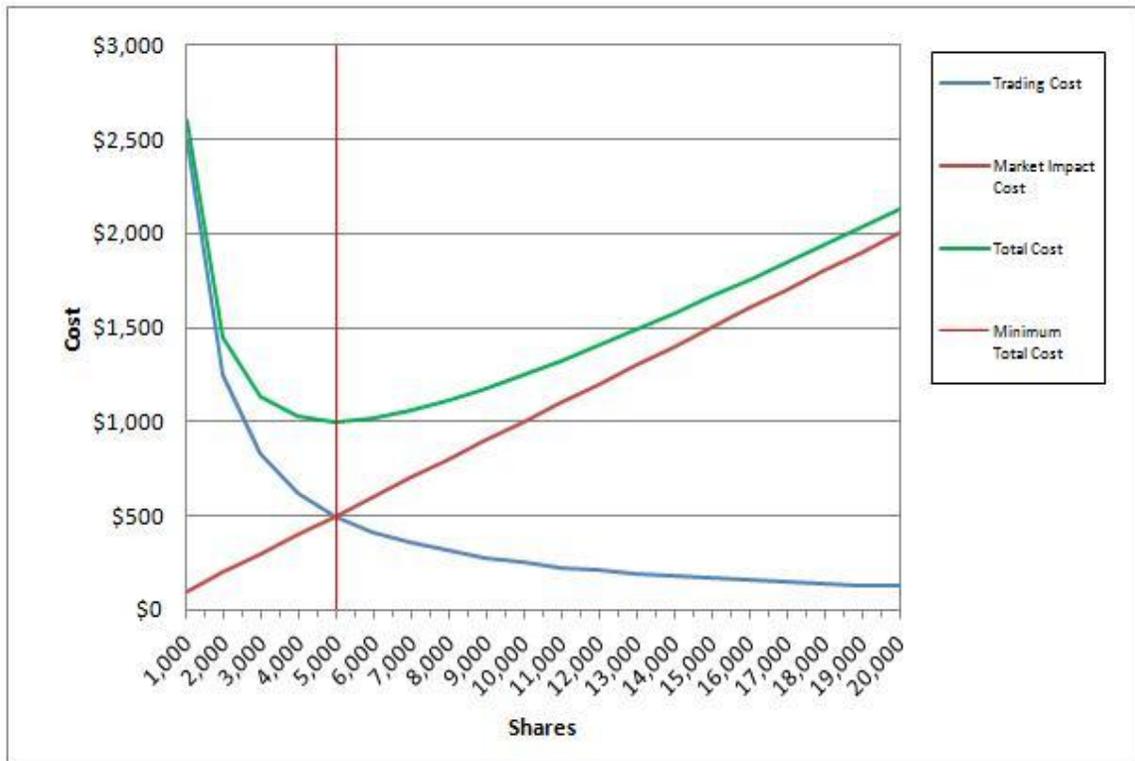
Figure 1: $\widehat{EOQ}$ when market impact cost is fixed.

Notice that the minimum occurs where trading cost equals market impact cost, which as expected is at the point where the minimum total cost occurs.

Now let's repeat the same exercise as above, but this time assuming market impact cost is variable, is a function of shares traded, and of the form $a \cdot \hat{Q}^b$. These details are described in Table 2.

Table 2	
Company:	IBM
Average Daily Volume:	4,500,000
Total Shares to Execute:	1,500,000
Parameter a:	0.085
Parameter b:	1.125
Market Impact Cost/Share:	$a \cdot \hat{Q}^b$
Fixed Cost/Order:	$2.50
Average Price/Share:	$172.00

The difficulty now is that market impact cost per share is no longer a constant, so trying to set trading cost equal to market impact cost, which had worked previously, won't work now. After differentiating $\widehat{TC}$ one will in general not be able one to combine the $\hat{Q}$ terms and proceed to solve for $\widehat{EOQ}$ as before. Thus what is needed is some type of optimization scheme[7]. Solving via optimization it turns out that the optimal order quantity, $\widehat{EOQ}$, is 4,287. We can see this graphically in Figure 2.

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[7]The Nelder-Mead Simplex and Newton-Raphson methods (applied to the $\widehat{TC}$ formula) can be used.
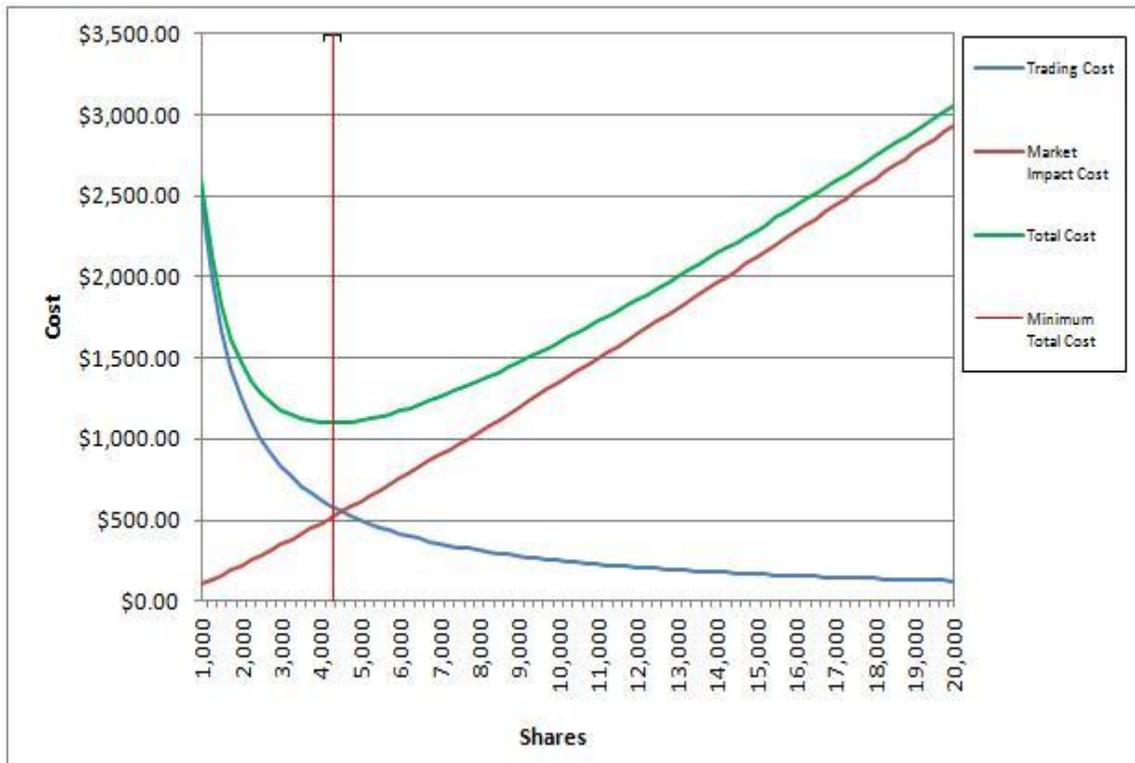
Figure 2: $\widehat{EOQ}$ when market impact cost is variable.

Note that as previously mentioned, $\widehat{EOQ}$ did not occur where trading cost equaled market impact cost. This was due to the nonlinearity of the market impact cost function.

5   Conclusion

As mentioned previously, trading cost such as commissions are easily known in advance, but market impact cost will have to be estimated separately and depending on approach taken can be quite complex to model. However, once the functional form has been determined it can easily be inserted into the application of $\widehat{EOQ}$ straightforwardly. Thus, for those seeking a practical and computationally easy way to calculate total execution cost for an equity trading program, the modified Economic Order Quantity model should be considered.